$\int^{az} u\, dV$. If concentration is correlated with velocity, the two are different: the latter is the concentration in the material balance equations, the former is nearer the correct value to be used in mass transfer rate calculations. This stagnancy effect has been held to account for the significant difference between mass transfer rates in absorption-desorption and vaporization experiments. It is usually dealt with by assuming a reduced effective interfacial area. It appears to produce no effects which depend upon column length.

Forward and backmixing have not been separately measured in tracer tests. But it has been shown that the dispersion can be explained, and with better success, by models other than dispersed plug flow (*11*, *12*).

Brittan (*13*) concluded that under ordinary conditions backmixing has only a slight influence on mass transfer efficiency. That conclusion is endorsed. It would appear that the end effects found in the work of Yoshida et al. should be ascribed, instead, to the rather crude liquid distribution system that was employed.

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NOTATION

c = concentration, lb./cu.ft.
G = gas rate, lb./(hr.)(sq.ft.)
L = liquid rate, lb./(hr.)(sq.ft.)
u = velocity, ft./sec.
V = volume of either phase, cu.ft.
z = height of column, ft.
y = mole fraction of solute in gas phase
y' = gas-phase mole fraction in equilibrium with liquid phase

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Compressible Isothermal Slip Flow in the Entrance Region

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Flow development in the hydrodynamic entrance region of a tube and a parallel plate channel in slip-flow regime has been given attention in the recent years. This problem is, in general, treated as an extension of continuum flow with slip at the wall. Sparrow et al. (*1*) linearized the momentum equation with the methods of Langharr and Targ and solved the channel flow, while Hanks (*2*) solved the tube flow with Targ's linearization. Quarmby (*3*) applied an improved linearization by equating the pressure drop based on momentum to that from mechanical energy considerations and solved for both tube and channel. A finite-difference analysis of the boundary-layer equation has also been carried out by Quarmby (*4*). Chen (*5*) used a flat center core and a parabolic boundary-layer velocity profile to integrate the continuity, momentum, and mechanical energy equations and then solved the resulting differential equations numerically.

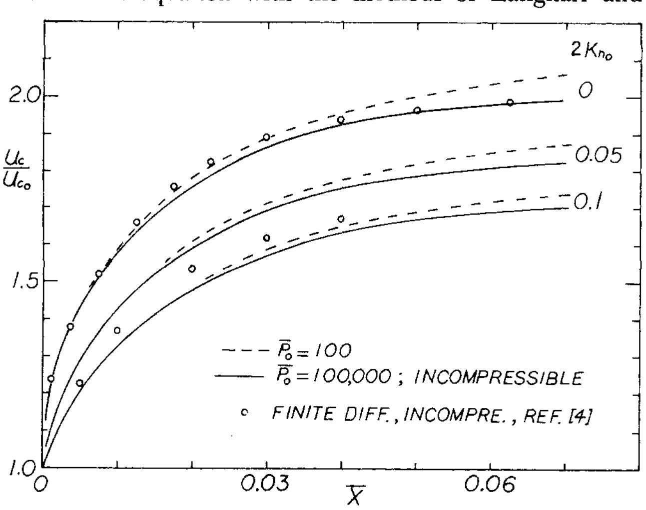

Fig. 1. Centerline velocity development in a tube.

The various analyses so far have been done for incompressible slip flow and are applicable to flow in which $\Delta p/p << 1$. In practical application, however, the pressure drop in the entrance region may be so large that the change in gas density cannot be neglected. In this work the integral method (*5*) is extended to include the effect of compressibility, and a solution for isothermal slip flow in the entrance region of a tube and a parallel plate channel is presented.

THEORETICAL ANALYSIS

Entrance Region of a Tube

For steady flow through a circular tube the continuity and boundary layer momentum equations are

$$\frac{\partial(\rho u r)}{\partial x} + \frac{\partial(\rho v r)}{\partial r} = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\frac{dp}{dx} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r \frac{\partial u}{\partial x}\right) \qquad (2)$$

By integrating Equations (1), (2), and the mechanical energy equation, which is obtained through multiplying u to Equation (2), over the cross section of the tube, and observing the condition that at $r = a$, $v = 0$, one finds

$$\int_0^a \rho\, u\, r\, dr = \text{const.} \qquad (3)$$

$$\frac{d}{dx}\int_0^a 2\,\rho\, u^2\, r\, dr + a^2 \frac{dp}{dx} = 2\,\mu\, a \left.\frac{\partial u}{\partial r}\right|_{r=a} \qquad (4)$$

$$\frac{1}{2}\frac{d}{dx}\int_0^a \rho\, u^3\, r\, dr + \frac{dp}{dx}\int_0^a u\, r\, dr = -\mu \int_0^a \left(\frac{\partial u}{\partial r}\right)^2 r\, dr + \mu\, u\, a \left.\frac{\partial u}{\partial r}\right|_{r=a} \qquad (5)$$

Equations (4) and (5) are subject to the first-order velocity-slip boundary condition $u(x, a) = -2a\, Kn(\partial u/\partial r)_a$ in which the Knudsen number Kn is defined as $C_1\lambda/2a$.

Recently Sreekanth (6) measured pressure distribution of a fully developed isothermal flow in a long tube and showed that in the slip-flow regime there was excellent agreement between experiment and theory based on parabolic velocity profile with slip at the wall. The velocity profile in the present analysis is, therefore, assumed to consist of a flat center core and a parabolic profile in the boundary layer, as in reference 5. Equations (3) through (5) are nondimensionalized and then integrated with the assumed velocity profile to obtain

$$I = \int_0^1 \beta\, \bar{\rho}\, \bar{u}\, 2\, \bar{r}\, d\bar{r} = \alpha\, \beta\, f = \text{const.} \qquad (6)$$

$$\frac{d}{d\bar{x}}\left[\frac{2g}{\alpha f^2}\right] + \bar{p}_0 \frac{d\alpha}{d\bar{x}} = -\frac{32c}{\alpha f m} \qquad (7)$$

$$\frac{d}{d\bar{x}}\left[\frac{h}{\alpha^2 f^3}\right] + \frac{\bar{p}_0}{\alpha}\frac{d\alpha}{d\bar{x}} = -\frac{16c(6 - 2c - cm)}{3\, m\, \alpha^2 f^2} \qquad (8)$$

where $c = m\alpha/(m\alpha + 4\, Kn_0)$ due to the fact that for isothermal flow $Kn\, p = Kn_0\, p_0$. The functions f, g, h are listed in reference 5.

After carrying out the differentiation on the left-hand side of Equations (7) and (8), one obtains a system of first-order differential equations with m and α as independent variables. Numerical solutions of Equations (6) through (8) for $m = 0$ at $\bar{x} = 0$ are presented in Figure 1 for u_c/u_{c0} and in Figure 2 for $K(\bar{x})$, which is the excess pressure drop over the incompressible fully developed flow.

Entrance Region of a Parallel Plate Channel

The channel is formed by two parallel plates spaced at a distance of $2\,a$ and the x coordinate of a Cartesian system is placed along the centerline of the channel. By using the same approach as in the case of tube flow, one can obtain equations corresponding to Equations (6) through (8):

$$J = \int_0^1 \beta \bar{\rho}\, \bar{u}\, d\bar{y} = \alpha\beta F \qquad (6a)$$

$$\frac{d}{d\bar{x}}\left[\frac{2\,G}{\alpha F^2}\right] + \bar{p}_0 \frac{d\alpha}{d\bar{x}} = -\frac{16\,c}{\alpha m\, F} \qquad (7a)$$

$$\frac{d}{d\bar{x}}\left[\frac{H}{\alpha^2 F^3}\right] + \frac{\bar{p}_0}{\alpha}\frac{d\alpha}{d\bar{x}} = -\frac{16\,c\,(3 - c)}{3\,\alpha^2 m\, F^2} \qquad (8a)$$

$$F = \int_0^1 \bar{u}\, d\bar{y} = (3 - c\, m)/3 \qquad (9)$$

$$G = \int_0^1 \bar{u}^2\, d\bar{y} = (15 - 10\, c\, m + 3\, c^2 m)/15 \qquad (10)$$

$$H = \int_0^1 \bar{u}^3\, d\bar{y} = (35 - 35\, c\, m + 21\, c^2\, m - 5\, c^3\, m)/35 \qquad (11)$$

The factor c is the same as in tube flow.

Numerical solutions of Equations (6a) through (8a) for $m = 0$ at $\bar{x} = 0$ are shown in Figures 3 and 4. In Figure 4 K_p is the excess pressure drop over the fully de-

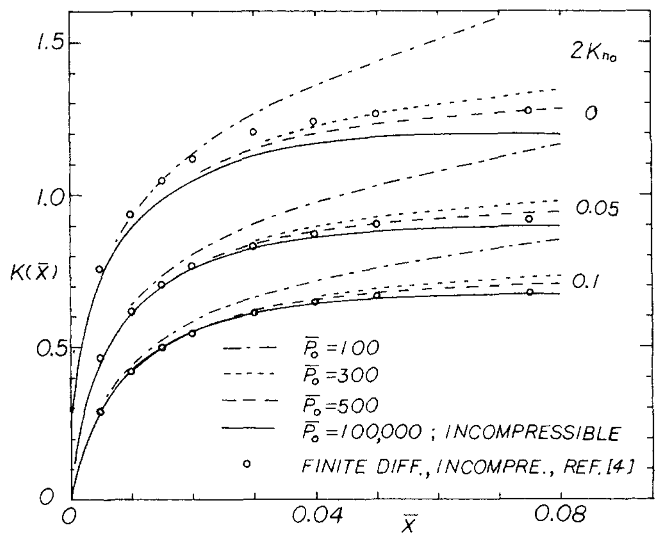

Fig. 2. Excess pressure drop in a tube.

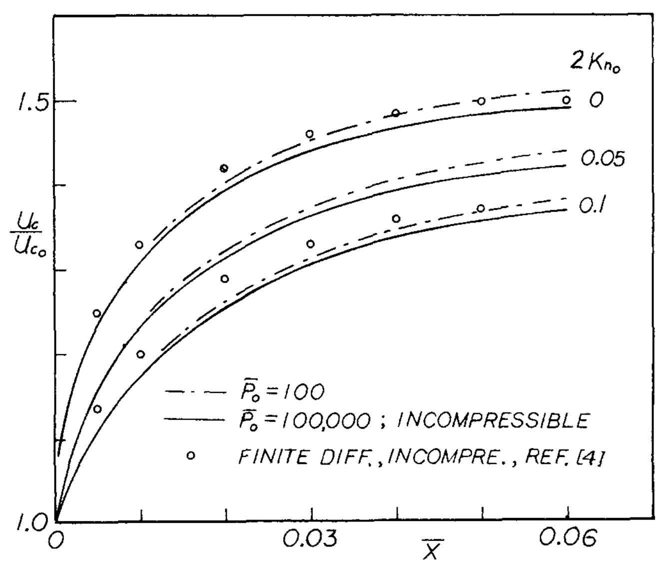

Fig. 3. Centerline velocity development in a parallel plate channel.

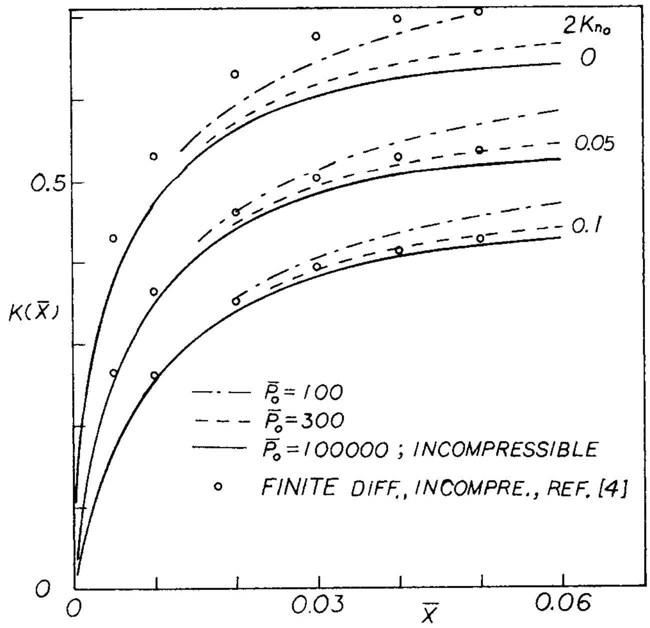

Fig. 4. Excess pressure drop in a parallel plate channel.

veloped incompressible flow in a parallel plate channel of $24\,\bar{x}/(1 + 6\,Kn_0)$.

RESULTS AND DISCUSSION

The present analysis reduces to incompressible flow when $\bar{p}_0$, which is $2/kM_0^2$, approaches infinity, or say 10^5, which is corresponding to an inlet Mach number of 0.00378 for $k = 1.40$. As seen from Figures 1 through 4, the centerline velocity development is less sensitive to the effect of compressibility than the excess pressure drop function. A flow in a tube with $\bar{p}_0 = 300$ (that is, $M_0 = 0.069$ for $k = 1.40$) gives about 2% (2% for channel) higher centerline velocity and 8% (5% for channel) higher excess pressure drop than that of incompressible flow. However, the difference in the total pressure drop is less than 3% (1% for channel). The analysis also shows that the boundary-layer growth of an isothermal flow is slightly slower than that of incompressible flow.

If the boundary-layer thickness at the inlet plane of a low-density gas flow in a tube is known, the present theory may be used to compare the experimental result. Unfortunately, the writer is not aware of any experimental investigation on slip flow in the entrance region.

Comparison of existing incompressible flow analyses shows that the present solution gives fair description of the centerline velocity development and best description on excess pressure drop, except that obtained through finite-difference analysis (*4*). To increase the range of applicability in the slip-flow regime, one may include the second-order slip-velocity boundary conditions (*6*).

The effects of Reynolds number and axial shear on the entrance length and velocity development have been shown by several authors (*7, 8*) for continuum flow regime. These effects are being investigated. It is felt that these effects would be less in the slip-flow regime due to slip at the wall. A further extension of the present work would be flow in a tube with adiabatic wall. For such flow one could assume an isoenergetic (homenergic) flow—a flow with adiabatic wall and a Prandtl number of unity. As in one-dimensional flow one will expect that isothermal flow gives higher pressure drop than flow with adiabatic wall.

CONCLUSIONS

The effects of compressibility on the entrance flow have been analyzed with integral momentum methods. The results show that

1. For flow of $\bar{p}_0$ ($= 2/kM_0^2$) greater than 300, incompressible flow analysis gives satisfactory results.
2. The centerline velocity development is less sensitive to the compressibility effect than the excess pressure drop.
3. Slower growth in the boundary layer for isothermal compressible flow than that for incompressible flow is observed.

NOTATION

a = radius of tube or half-width of channel
c = slip factor, $m\alpha/(m\alpha + 4Kn_0)$
C_1 = first-order slip coefficient
F, G, H defined in Equations (9) through (11)
f, g, h defined in reference 5
I defined in Equation (6)
J defined in Equation (6a)
K = excess pressure drop for tube
Kn = Knudsen number, $C_1\lambda/2a$
K_p = excess pressure drop for channel
k = specific heats ratio
M_0 = Mach number at inlet plane based on V_0
m = boundary-layer thickness/a
p = static pressure
$\bar{p}$ = dimensionless pressure, $2p/(\rho_0 V_0^2)$
r = radial distance
$\bar{r}$ = dimensionless radial distance, r/a
N_{Re} = Reynolds number, $2a\rho_0 V_0/\mu$
u = velocity in x direction
$\bar{u}$ = dimensionless velocity, u/u_c
u_c = centerline velocity in x direction
u_{c0} = centerline velocity at inlet
V_0 = average velocity at inlet
v = velocity in r direction
x = coordinate in the direction of flow
$\bar{x}$ = dimensionless coordinate $x/(2a\,N_{Re})$
y = coordinate normal to flow
$\bar{y}$ = dimensionless coordinate, y/a

Greek Letters

α = pressure ratio, p/p_0
β = centerline velocity ratio, u_c/u_{c0}
ρ = density
λ = molecular mean free path
μ = viscosity

Subscripts

0 = inlet conditions
c = centerline conditions

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